Theorem-Prover based Systems (TPS) for Education (eduTPS)

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This group continues work begun at CADGME'09/10 entitled "Convergence on Mathematics Assistants". Since then, a framework for such convergence has been identified: automated and interactive computer theorem proving, appropriate both as a conceptional framework and a source for supportive technology. The group will discuss the potential of Theorem-Prover based Systems (TPS) for educational practice.

Recently and largely unnoticed in public, applications in science and technology drove the development of automated and interactive theorem proving technologies, which have become of major importance for mathematics and computer science in academia and in industry. However, their potential for a wide-spread education technology is unexplored, in spite of the fact, that TPS exhibit features relevant for mathematics education:

1. **TPS check user-input automatically**: since input states a lemma to be proved within the logical context of a proof, a calculation or a geometric construction, TPS check user-input without code specific for a large class of input. This automated support for interactivity brings TPS for step-wise problem solving within reach.

2. **TPS cover the whole problem solving process**: since TPS implement reasoning – the core of mathematical thinking technology, they support all steps in problem solving (mathematising, comparing representations, reasoning and argumenting, trying various strategies, using formal language until a solution can be verified) — TPS are "complete" in this sense.

3. **TPS are self-explaining models of mathematics**: since underlying math knowledge is represented down to "first principles" (i.e. beginning from the definition of set) in a human readable format, this can be made transparent to learners; so, being transparent, complete and interactive, self-explaining TPS come within reach.

These features are distinguished from present educational mathematics software, from CAS, DGS, Spreadsheets etc. such that they establish a new generation of educational math assistants. Several prototypes for this generation are under construction in academic R&D [1] [2] for geometry, algebra and applications in various engineering disciplines. Some of these prototypes will improve usability and/or will be integrated into existing educational tools within the next years. So it seems in time to discuss TPS' potential and expected impact on educational practice:

[1] http://www.uc.pt/en/congressos/thedu/thedu11/

[2] http://www.informatik.uni-bremen.de/cicm2012/cicm.php?event=thedu&menu=general

- What are the novel promises of TPS for open learning scenarios in class, independent learning at home, in renewed math and science education?

- How can TPS provide additional challenges for gifted and interested students as well as extra tuition to catch up on, particularly for ”slow but rigorous thinkers”?

- As ”self-explaining models of math”, can TPS establish learning in math as is possible with a chess program: do some moves (steps in problem solving), if the situation becomes hopeless, backtrack to a previous situation and try other moves (the ”game of math” is won, if a solution of a problem has been reached)?

- What is the gain for designing curricula, when respective math knowledge can be mechanized and is available ”from first principles”?

- What is the gain for evaluation and assessment, when the same software can be used for learning as well as for assessment (because for the latter only supportive functionality needs to be reduced)?

- How can TPS support and enforce continuity between intuitive math at high-school and formal math at university ?

- What are novel applications in math wikis (making proofs and example calculations interactive), in museums, math spaces etc. (interactively present math as a key thinking technology)?

- Are there ideas for open price competitions addressing the public in interactive mathematical challenges via cloud computing?

- ... etc.